11.4

POINT (x_0, y_0, z_0)

DIRECTION VECTOR (PARALLEL TO LINE) $\langle a, b, c \rangle$

OR ANY SCALAR MULTIPLE (EXCEPT $0\vec{v}$)

PARAMETRIC EQUATIONS

$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

SYMMETRIC EQUATIONS

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

$(a, b, c \neq 0)$

eg. FIND SYM EQ'NS FOR LINE THROUGH $(2, 5, -4)$ WITH DIR VEC $\langle -1, 1, 0 \rangle$

?

$$\cancel{\frac{x-2}{-1} = \frac{y-5}{1} = \frac{z+4}{0}}$$

$$\frac{x-2}{-1} = \frac{y-5}{1}, \quad z = -4$$

$$\boxed{2 - x = y - 5, \quad z = -4}$$

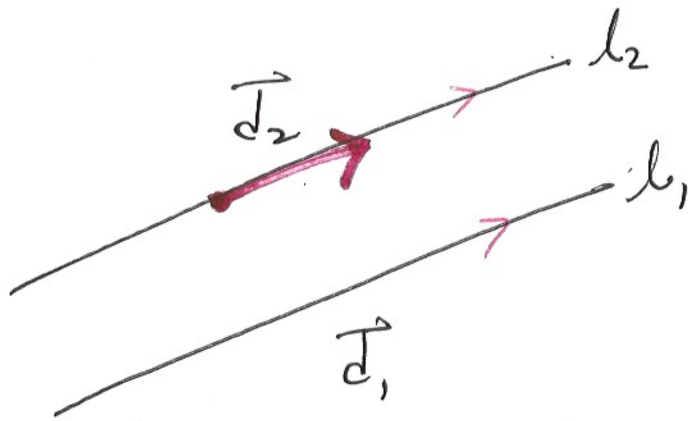
$x = 2 - 1t = 2 - t \qquad t = \frac{x-2}{-1}$

$y = 5 + 1t = 5 + t \rightarrow t = \frac{y-5}{1}$

$z = -4 + 0t = -4$

FIND SYM EQ'NS OF LINE THROUGH $(0,-2,5)$ $\Big\}\ l_1$

AND PARALLEL TO $\left.\begin{aligned} x &= 6-3t \\ y &= t+5 \\ z &= 4 \end{aligned}\right\}\ l_2$



$$\vec{d}_1 = \vec{d}_2 = \langle -3, 1, 0 \rangle$$

$$\frac{x-0}{-3} = \frac{y--2}{1} = \cancel{\frac{z-5}{0}}$$

$$-\frac{x}{3} = y+2, \; z=5$$

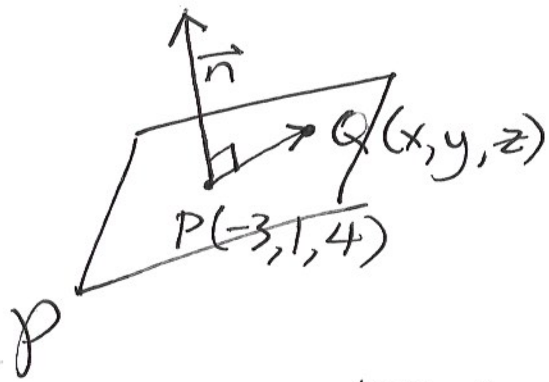
IN ORDER TO SPECIFY A PLANE,

YOU NEED A POINT ON THE PLANE

+ A VECTOR PERPENDICULAR TO THE PLANE

↳ A/K/A NORMAL VECTOR

eg. FIND AN EQUATION OF THE PLANE THROUGH $P(-3,1,4)$ WITH NORMAL VECTOR $\vec{n} = \langle 5,-2,7\rangle$



LET $Q(x,y,z)$ BE ON THE PLANE

$$\overrightarrow{PQ} \perp \vec{n} \xrightarrow[\text{CONCEPT}]{\text{CRITICAL}} \overrightarrow{PQ}\cdot\vec{n} = 0$$

$$\langle x-(-3), y-1, z-4\rangle \cdot \langle 5,-2,7\rangle = 0$$

$$5(x+3)-2(y-1)+7(z-4)=0$$

POINT-NORMAL FORM

OR

$$5x+15-2y+2+7z-28=0$$

$$5x-2y+7z-11=0$$

SIMPLIFIED FORM

IN GENERAL,
THE PLANE THROUGH (x_0, y_0, z_0)
WITH NORMAL VECTOR $\langle A, B, C\rangle$
HAS POINT-NORMAL EQUATION

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

AND SIMPLIFIED EQUATION

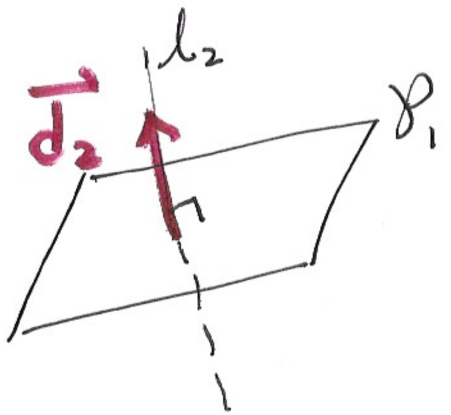
$$Ax+By+Cz-(Ax_0+By_0+Cz_0)=0$$

$$Ax+By+Cz+D=0 \qquad D=-(Ax_0+By_0+Cz_0)$$

FIND THE POINT-NORMAL FORM OF THE EQUATION OF THE PLANE THROUGH $(-5,0,1) \leftarrow P_1$ AND PERPENDICULAR TO $\underbrace{x+2=\frac{4-y}{2}=z}_{\text{SYM EQ'N OF LINE}} \leftarrow \ell_2$

$$\frac{x-(-2)}{1}=\frac{y-4}{-2}=$$

$\vec{d}_2 = \langle 1,-2,1\rangle$



NEED $\vec{n}_1 \perp$ PLANE

USE $\vec{n}_1 = \vec{d}_2 = \langle 1,-2,1\rangle$

SINCE $\vec{d}_2 \parallel \ell_2$

AND $\ell_2 \perp P_1$

THEREFORE $\vec{d}_2 \perp P_1$

$$1(x- -5) -2(y-0)+1(z-1)=0$$

$$1(x+5)-2(y-0)+1(z-1)=0$$

$$(x+5)-2y+(z-1)=0$$

FIND THE SIMPLIFIED EQ'N OF THE PLANE THROUGH $(-4,2,-5)$ $\Big\}\ P_1$

AND PARALLEL TO

$$\underbrace{\begin{aligned} x &= 5t-4 \\ y &= 1+2t \\ z &= 3-t \end{aligned}}_{\text{PARAMETRIC EQ'NS OF A LINE } \ell_2}$$

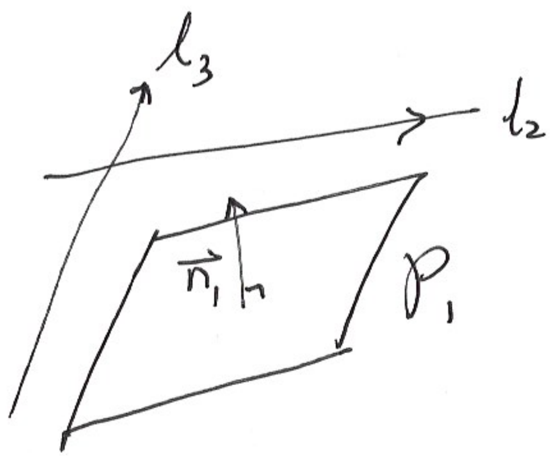
PARAMETRIC EQ'NS OF A LINE $\ell_2 \to \vec{d_2} = \langle 5, 2, -1 \rangle$

AND ALSO

$$\underbrace{x = 1,\ \frac{y+2}{3} = -z}_{\text{SYMMETRIC EQ'NS OF A LINE } \ell_3}$$

SYMMETRIC EQ'NS OF A LINE $\ell_3 \to$

$\vec{d_3} = \langle 0, 3, -1 \rangle$



$\vec{n_1} \perp \vec{d_2}, \vec{d_3}$

LET $\vec{n_1} = \vec{d_2} \times \vec{d_3}$

$$\vec{n_1} = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 5 & 2 & -1 & 5 & 2 \\ 0 & 3 & -1 & 0 & 3 \end{array}\right.$$

MANDATORY SANITY CHECK!